Absorptive and dispersive properties in the phase-sensitive optical parametric amplification inside a cavity

Chenguang Ye and Jing Zhang*

The State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan 030006, People's Republic of China

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In this paper we investigate the absorptive and dispersive properties in the phase-sensitive optical parametric amplification inside a cavity. First, we study the single-resonant optical parametric amplifier theoretically and experimentally. In this case, the mode splitting in the transmission spectra and the M shape in the reflection spectra are observed. However, the shape of the phase shift of the reflected field is unchanged. Then, the double-resonant optical parametric amplifier is studied theoretically, in which electromagnetically induced transparency-like effect may be emulated when the cavity linewidth for the harmonic wave is narrower than for the subharmonic field. The narrow transparency window appears in the absorption spectrum and is accompanied by a very steep variation of the dispersive profile.

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I. INTRODUCTION

Coherence and interference effects play important roles in determining the optical properties of quantum systems. Electromagnetically induced transparency [1,2] (EIT) in quantum-mechanical atomic systems is a well understood and thoroughly studied subject. EIT has been utilized in a variety of applications, such as lasing without inversion [3], slow and stored light [4–8], enhanced nonlinear optics [9–12], and quantum computation and communication [13–15]. Relying on destructive quantum interference, EIT is a phenomenon where the absorption of a probe laser field resonant with an atomic transition is reduced or even eliminated by the application of a strong driving laser to an adjacent transition. Since EIT results from destructive quantum interference, it has been recently recognized that similar coherence and interference effects also occur in classical systems, such as plasma [16–18], coupled optical resonators [19–24], and mechanical or electric oscillators [25,26]. The phenomenology of the EIT and dynamic Stark effect is studied theoretically in a dissipative system composed by two coupled oscillators under linear and parametric amplification using quantum optics model in Ref. [27]. Recently, we observed mode splitting in transmission profile of a phase-sensitive optical parametric amplifier (OPA) experimentally [28]. This phenomenon results from the interference between the harmonic pump field and the subharmonic seed field in an OPA. The destructive and constructive interference correspond to optical parametric deamplifier and amplifier, respectively, which are in cooperation with dissipation of the cavity. The absorptive response of an optical cavity for the probe field is changed by optical parametric interaction in the cavity.

In our previous work, we only measured the transmission profile of single-resonant an OPA, in which the subharmonic field is resonating inside the cavity and harmonic wave makes a double pass through the cavity. In this paper, we investigate coherence effects of the OPA in detail. First, we present the absorptive and dispersive response of the reflection of a single-resonant OPA at deamplification for the subharmonic field. It is shown that the absorption profile is significant changed and presents "M" shape, however, the dispersion profile still keeps unchanging. Then we investigate theoretically the absorptive and dispersive response of a double-resonant OPA at deamplification, in which both the subharmonic field and harmonic wave resonate simutaneously inside the cavity. EIT-like effect may be emulated in an OPA when the cavity linewidth for the harmonic wave is narrower than for the subharmonic field. The narrow transparency window appears in the absorption spectrum and is accompanied by a very steep variation of the dispersive profile.

II. THE ABSORPTIVE AND DISPERSIVE RESPONSE IN A SINGLE-RESONANT OPA

Consider the interaction of two optical fields of frequencies ω and 2ω, denoted by a subharmonic (the probe) and harmonic wave (the pump), which are coupled by a second-order, type-I nonlinear crystal in a optical cavity as shown in Fig. 1. The cavity is assumed to be a standing wave cavity length L (roundtrip time $\tau = 2L/c$), and only resonant for the subharmonic field with a dual-port of the transmission T_{ref}

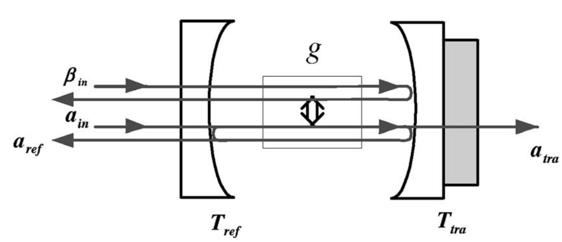

FIG. 1. Schematic of optical parametric amplifier inside standing-wave cavity.

*Corresponding author. Email address: jzhang74@yahoo.com

and T_{tra}, and internal losses A. We consider both the subharmonic seed beam a^{in} and the harmonic pump beam β^{in} are injected into the backport (T_{ref} mirror) of the cavity, where the relative phase between the injected fields is adjusted by a movable mirror out of the cavity. The T_{ref} mirror is a high reflectivity mirror at the subharmonic wavelength, yet has a high transmission coefficient at the harmonic wavelength and T_{tra} mirror has a high reflectivity coefficient for the harmonic wave. The harmonic wave makes a double pass through the nonlinear medium. The equation of motion for the mean value of the subharmonic intracavity field can then be derived by the semiclassical method as [29]

$$\tau\frac{da}{dt} = -i\Delta\tau a - \gamma a + g\beta^{in}a^{*} + \sqrt{2\gamma_{in}}a^{in}. \tag{1}$$

Here the decay rate for internal losses is $\gamma_l = A/2$ and the dampings associated with coupling mirror and backmirror are $\gamma_c = T_{tra}/2$ and $\gamma_{in} = T_{ref}/2$, respectively. The total damping is denoted by $\gamma = \gamma_{in} + \gamma_c + \gamma_l$. Δ is the detuning between the cavity-resonance frequency ω_c and the subharmonic field frequency ω. The strength of the interaction is characterized by the nonlinear coupling parameter g. Equation (1) is complemented with the boundary conditions $a^{tra} = \sqrt{2\gamma_c}a$ and $a^{ref} = -a^{in} + \sqrt{2\gamma_{in}}a$ creating propagating beams, where a^{tra} is the transmitted field from the coupling mirror T_{tra} and a^{ref} is the reflected field from the backmirror T_{ref}. The phase-sensitive optical parametric amplifier always operates below the optical parametric oscillation (OPO) threshold $\beta^{in}_{th} = \gamma/g$. Here, Eq. (1) ignores the third-order term [30] describing the conversion losses due to harmonic generation. For simplicity, we assume that the phase of the pump field is zero in the discussion, i.e, β^{in} is real and positive. The mean values of the intracavity field a and the injected field a^{in} are expressed as $\langle a\rangle = \alpha\exp(-i\phi)$ and $\langle a^{in}\rangle = A_{in}\exp(-i\varphi)$, respectively. Here, α and A_{in} are real, ϕ and φ are the relative phase between the intracavity field and the pump field and between the seed field and the pump field, respectively. If the harmonic pump is turned off, the throughput for the nonimpedance matched subharmonic seed beam is given by $\langle a^{tra}_{no\ pump}\rangle = 2\sqrt{\gamma_c\gamma_{in}}A_{in}/(\gamma + i\tau\Delta)$. The subharmonic seed beam is subjected to either amplification or deamplification, depending on the chosen relative phase between the subharmonic field and the pump field.

Consider the transmitted intensity of the subharmonic seed beam as a function of the detuning Δ between the subharmonic field frequency and the cavity-resonance frequency, and keep the pump field of frequency $\omega_p = 2\omega$ constant. Setting the derivative to zero ($da/dt = 0$) and separating the real and image part of Eq. (1), the steady state solutions of the amplitude and relative phase of the intra-cavity field are given by

$$-\gamma\alpha + g\beta^{in}\alpha\cos 2\phi + \sqrt{2\gamma_{in}}A_{in}\cos(\phi - \varphi) = 0,$$

$$-\tau\Delta\alpha + g\beta^{in}\alpha\sin 2\phi + \sqrt{2\gamma_{in}}A_{in}\sin(\phi - \varphi) = 0. \tag{2}$$

When the amplitude and relative phase of the subharmonic seed beam are given, the reflected and transmitted fields of the subharmonic beam are obtained from Eq. (2) and the boundary condition. Figures 2(a)–2(c) show the amplitude of the subharmonic reflection, transmission, and phase shift of the reflected field when the pump field is absent. This corresponds to the typical absorptive and dispersive response of the optical empty cavity. The transmitted spectrum presents a Lorentzian profile. When the injected subharmonic field is out of phase ($\varphi = \pi/2$) with the pump field, the amplitude of the subharmonic reflection, transmission and phase shift are shown in Fig. 2(e)–2(g). There is a symmetric mode splitting in transmission profile and the reflection profile presents M shape. The origin of mode splitting in transmission spectra of the OPA is destructive interference in cooperation with dissipation of the cavity. If the subharmonic field is resonating in the cavity perfectly, i.e., $\Delta = 0$, the subharmonic intracavity field and the pump filed are exactly out of phase and they will interfere destructively to produce the deamplification for the subharmonic field in the nonlinear crystal. Thus a dip appears at the zero detuning of the transmission profile. If the subharmonic field does not quite resonate in the cavity perfectly, that is, the subharmonic field's frequency is not exactly an integer multiple of the free spectral range but close enough to buildup a standing wave, the phase difference between the subharmonic intracavity field and the pump field will not be exactly out of phase and will increase as the detuning is increased. The subharmonic intracavity field will

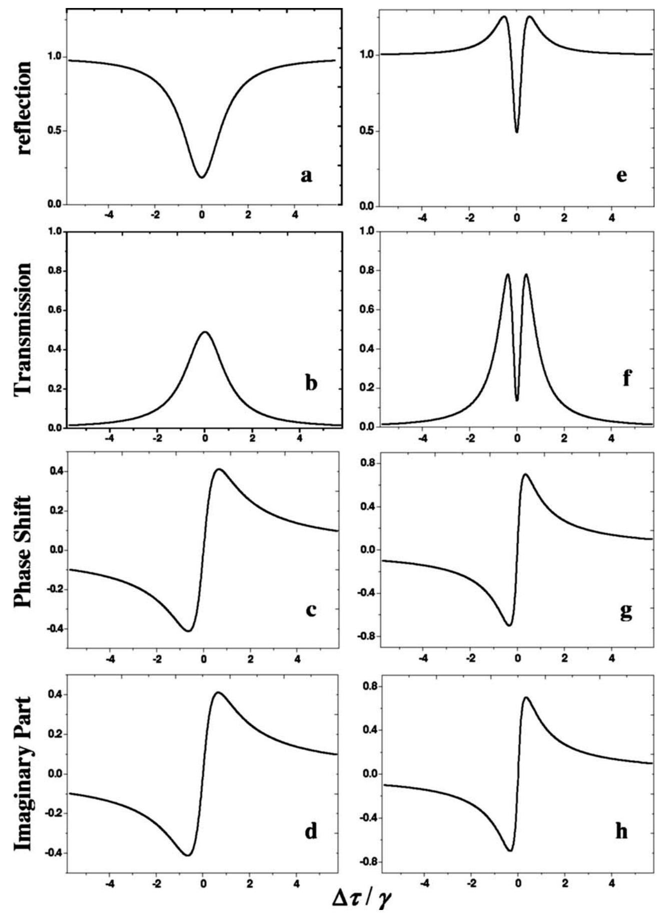

FIG. 2. The theoretical results for the single-resonant OPA: (a),(b),(c),(d) without the pump field injection and (e),(f),(g),(h) with $\beta^{in}/\beta^{in}_{th} = 0.9$.

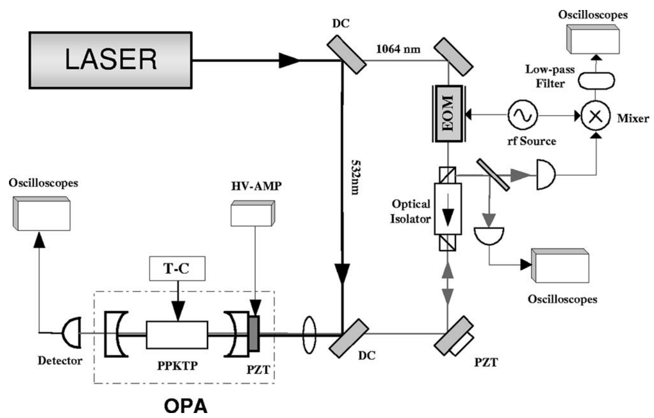

FIG. 3. Schematic of the experimental setup. DC: dichroic mirror; $\lambda/2$: half-wave plate; T-C, temperature controller, and HV-AMP: high voltage amplifier.

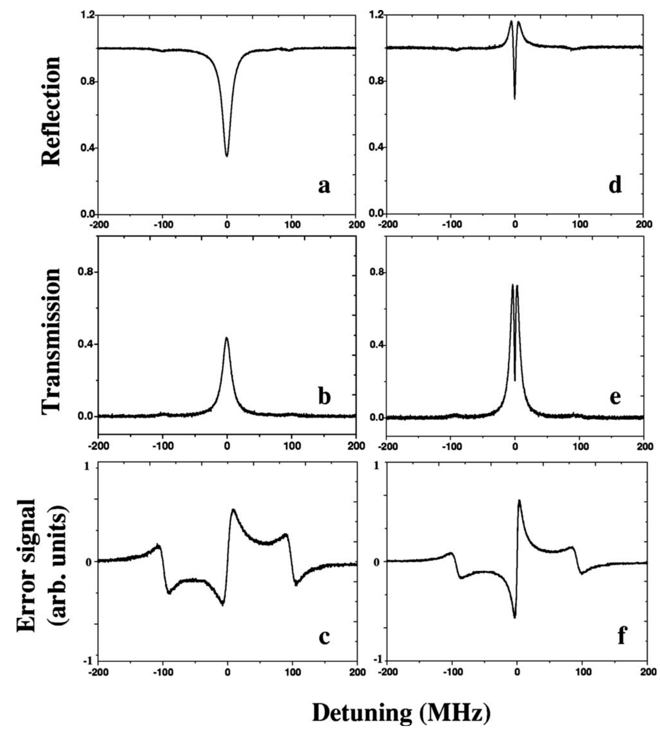

FIG. 4. The experimental results for the single-resonant OPA corresponding to Fig. 2.

change from deamplification to amplification when the phase difference increases. Thus we see that the transmission profile has two symmetric peaks at the detuning frequencies. The dip in the transmission profile becomes deeper and two peaks higher as the pump intensity is increased [28]. When the pump power is close to the threshold, two peaks of the transmission profile are higher than the intensities of the subharmonic field at zero detuning and without the pump. Correspondingly, the reflection profile appears two peaks near the threshold at the detuning frequencies, whose intensities are higher than that of the subharmonic seed beam. In our scheme, the cavity is an undercoupled resonator, whose dispersive response results in fast light [23,31]. The phase shifts of the reflected field are shown in Fig. 2(c) and 2(g) without pump, and with pump respectively. We see that the curve of the phase shift still keeps unchanging as the pump field is applied. The imaginary parts of the reflected field are shown in Figs. 2(d) and 2(h). When an undercoupled resonator is far from critically-coupled, the imaginary part of the reflected field approximates to the phase shift.

The experimental setup is shown schematically in Fig. 3. A diode-pumped intracavity frequency-doubled continuous-wave ring $Nd:YVO_4/KTP$ single-frequency green laser severs as the light sources of the pump wave (the second-harmonic wave at 532 nm) and the seed wave (the fundamental wave at 1064 nm) for the OPA. We actively control the relative phase between the subharmonic and the pump field by adjusting the phase of the subharmonic beam with a mirror mounted upon a piezoelectric transducer (PZT). Both beams are combined together by a dichroic mirror and then injected into the OPA cavity. The OPA consists of periodically poled $KTiOPO_4$ (PPKTP) crystal (12 mm long) and two external mirrors separated by 43 mm. Both end faces of the crystal are polished and coated with an antireflector for both wavelengths. The crystal is mounted in a copper block, whose temperature is actively controlled at precision of millidegrees kelvin level around the temperature for optical parametric process (31.3°C). The input coupler M1 is a 20 mm radius-of-curvature mirror with a power reflectivity 98.5% for 1064 nm in the concave and a total transmissivity 70% for 532 nm, which is mounted upon a PZT to adjust the cavity length. The output wave is extracted from M2, which is a 20 mm radius-of-curvature mirror with a total transmissivity 2% for 1064 nm and a reflectivity 99% for 532 nm in the concave. Due to the large transmission of the input coupler at 532 nm, the pump field can be thought as it only passes the cavity twice without resonance. Due to the high nonlinear coefficient of PPKTP, the measured threshold power is only 30 mW. In order to measure the phase of the reflected beam, we employ the Pound-Drever-Hall method (or frequency-modulation spectrum) [32–36], which provides us with a way of indirectly measuring the phase. The frequency of the injected subharmonic field is modulated with an electric-optic modulator (EOM), driven by a local oscillator with the frequency 100 MHz. The reflected beam is picked off with an optical isolator and sent into a photodetector, whose output is compared with the local oscillator's signal via a mixer. A low-pass filter on the output of the mixer extracts the low frequency signal, which is called the error signal. When the frequency of the injected subharmonic field is near with the resonance frequency and the modulation frequency is high enough such that the sidebands are out of the resonance, the error signal is the imaginary part of the reflected field.

Keeping the frequency of the subharmonic and the pump field with $\omega_p = 2\omega$, and the phase at $(\varphi = \pi/2)$ between the subharmonic field and the pump field, we scan the cavity length. Figure 4 shows the experimental results: (a), (b), and (c) are the amplitude of the subharmonic reflection, transmission, and error signal, respectively, without the pump field, (d), (e), and (f) that with $\beta^{in}/\beta^{in}_{th} = 0.9$. It can be seen that the experimental curves of the amplitude of the subharmonic reflection and transmission are in good agreement with the theoretical results shown in Fig. 2, which are obtained with the experimental parameters. The error signal near the zero detuning may approximate to the phase shift as shown in Figs. 2(c) and 2(f). We see that the shape of the error signal is unchanged as the pump field is applied.

III. THE ABSORPTIVE AND DISPERSIVE RESPONSE IN DOUBLE-RESONANT OPA

We consider the case of that the subharmonic and pump fields resonate inside the cavity simultaneously, and the linewidth of the pump field is narrower than that of the subharmonic field. The equations of motion for the mean values of the subharmonic and pump intracavity field are given by

$$\tau\frac{da}{dt} = -i\Delta\tau a - \gamma a + gba^* + \sqrt{2\gamma_{in}}a^{in},$$

$$\tau\frac{db}{dt} = -i\Delta\tau b - \gamma_b b + \sqrt{2\gamma_{in}^b}b^{in}, \tag{3}$$

where γ_{in}^b is the damping of the pump field due to the input mirror and γ_b is the total damping. The depletion of the pump is negligible since the OPA works as the deamplifier. Substituting the steady state solution $b=\sqrt{2\gamma_{in}^b}b^{in}/(i\Delta\tau+\gamma_b)$ of the pump field into the equation of the subharmonic field, we obtain

$$\tau\frac{da}{dt} = -i\Delta\tau a - \gamma a + g\frac{\sqrt{2\gamma_{in}^b}}{i\Delta\tau+\gamma_b}b^{in}a^* + \sqrt{2\gamma_{in}}a^{in}. \tag{4}$$

Comparing with Eq. (1), the third term describing the parametric coupling in Eq. (4) introduces the absorption and dispersion of the cavity for the pump field. The reflected and transmitted fields of the subharmonic beam are calculated from Eq. (4) and the boundary condition. Figure 5 shows the intensities of the subharmonic reflection, transmission and phase shift of the reflected field under deamplification at the different pump powers $b^{in}/b_{th}^{in}=0.26$ and $b^{in}/b_{th}^{in}=0.51$ respectively. Since the narrow absorption and dispersion of the pump field are introduced into the subharmonic field, the reflected field of the subharmonic field in OPA presents the EIT-like effect. The narrow transparency window appears in the absorption spectrum and is accompanied by a very steep variation of the dispersive profile. For the case of EIT, the condition analogous to $\gamma_b \ll \gamma$ is that a pair of lower energy states has long lifetime. As we increase the pump powers further, the intensities of the subharmonic reflection, transmission, and phase shift of the reflected field are shown in Fig. 6. The maximum absorption of the reflected field at zero detuning is $(\gamma-2\gamma_{in})^2/\gamma^2$ (without the pump). The intensity of the reflected field at zero detuning reaches a limited value $(\gamma-\gamma_{in})^2/\gamma^2$ when the pump power is near the threshold. However, the intensities of the reflected field for the nonzero detuning frequencies increase quickly and are higher than that at zero detuning. It is because the phase difference between the subharmonic intracavity field and the pump field will not be exactly out of phase at the detuning frequencies. The subharmonic intracavity field will change from deamplification to amplification as the phase difference increases. We see two transparency windows at the detuning frequencies, which are higher than that at zero detuning.

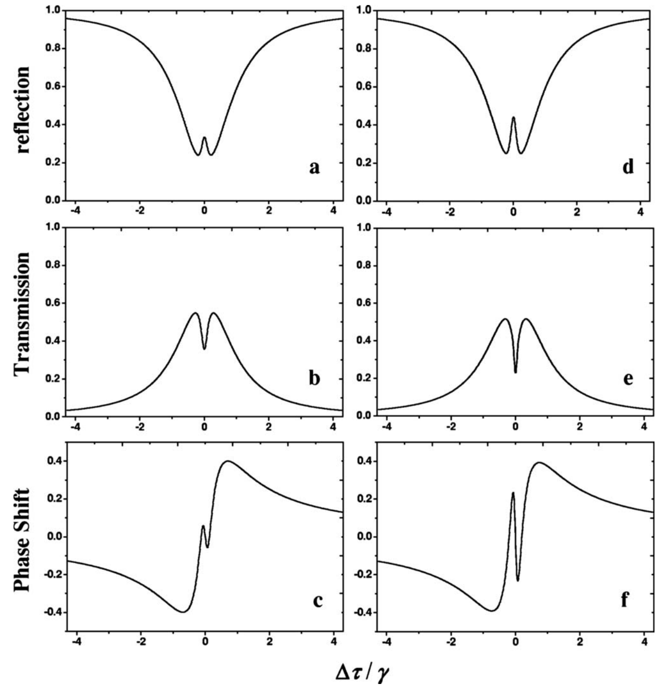

FIG. 5. The theoretical results for the double-resonant OPA: (a),(b),(c) with $b^{in}/b_{th}^{in}=0.26$ and (d),(e),(f) with $b^{in}/b_{th}^{in}=0.51$.

IV. CONCLUSION

We have investigated the absorptive and dispersive response of the reflection of the single-resonant and double-

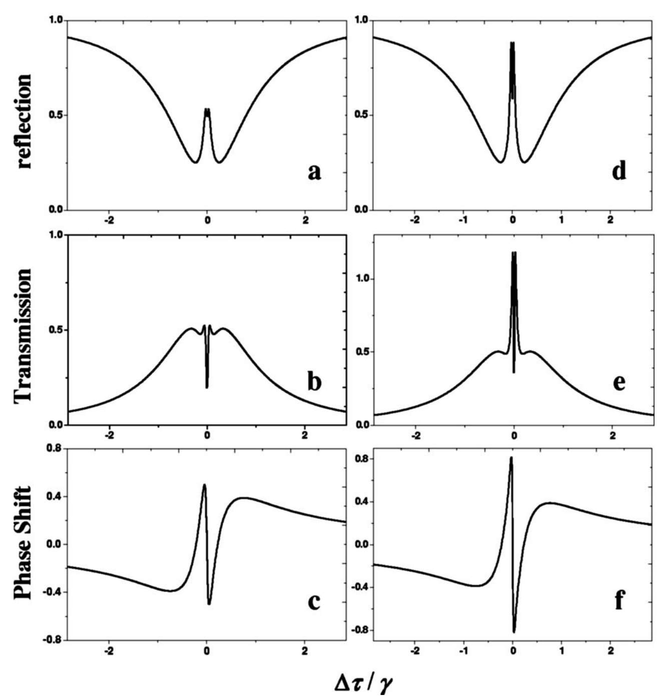

FIG. 6. The theoretical results for the double-resonant OPA: (a),(b),(c) with $b^{in}/b_{th}^{in}=0.64$ and (d),(e),(f) with $b^{in}/b_{th}^{in}=0.69$.

resonant phase-sensitive OPA in detail. For the case of the single-resonant OPA, the mode splitting in the transmission spectra and the M shape in the reflection spectra are observed. However, the shape of the phase shift of the reflected field is unchanged. For the case of the double-resonant OPA, an EIT-like effect is emulated when the cavity linewidth for the harmonic wave is narrower than for the subharmonic field. The narrow transparency window appears in the absorption spectrum and is accompanied by a very steep variation of the dispersive profile. This system will be important for practical optical and photonic applications such as optical filters, delay lines, and closely relate to the coherent phenomenon of EIT predicted for quantum systems.

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